TIME DOMAIN ELECTROMAGNETIC TOMOGRAPHY USING PROPAGATION AND BACKPROPAGATION METHOD

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ABSTRACT
This paper presents a new time domain electromagnetic tomographic imaging algorithm using the propagation and back propagation method by solving a time dependent Maxwell’s equation. The proposed algorithm reconstructs an extended object immersed in inhomogeneous medium using wide band electromagnetic pulse excitation. In this paper we develop an iterative yet convergent algorithm for reconstructing the dielectric profiles of the scattering medium for electromagnetic tomographic imaging purposes. The imaging algorithm employs the finite difference time domain forward propagation solver. The backpropagation solver iteratively reconstructs the object of interest. Using numerical examples, we demonstrate that the proposed algorithm results in high quality images with fast convergence.

Index Terms— Tomographic imaging, propagation and back propagation, iterative method

1. INTRODUCTION
The electromagnetic tomographic imaging technique has been widely used in many fields and applications. Some examples are breast cancer detection in biomedical imaging, subsurface sensing in geoscience, and land mine detection. In a typical microwave tomography system, an antenna array is used to transmit microwave signals into an object-under-test and receive scattered signals. By iteratively solving a nonlinear inverse problem, the dielectric properties of the object are quantitatively reconstructed from the received signals [1, 2]. The tomographic imaging technique differs from the the radar like backscattering methods for imaging. The radar scattering method does not seek the dielectric profile of the target but rather attempts to identify strong scatterers from their scattering medium. Tomographic image reconstruction can be performed based on mono-frequency data, multiple frequency data, [3] or time domain data [1]. Compared with frequency domain imaging systems, a wideband time domain imaging system has the advantage of fast data acquisition of broad frequency band data and low cost, which is preferable for electromagnetic tomography.

In this paper, we develop a time domain tomographic image reconstruction algorithm using the propagation and backpropagation (PBP) method. The tomographic imaging is formulated as a nonlinear inverse problem. The underlying wave propagation is described by the time dependent Maxwell’s equations. Solutions to the time dependent Maxwell’s equations in a general form are unknown except for a few special cases. Thus, computational methods are often used. In the PBP method, the forward propagation solver is based on the finite difference time domain (FDTD) computational technique [4]. The inverse problem is to reconstruct the dielectric properties of the target and is solved by the backpropagation method. The propagation and backpropagation method was first developed by Natterer for ultrasonic imaging applications [5]. We developed a multiple-input multiple-output (MIMO) ultrasonic imaging method in [6, 7]. Vogeler extended the PBP method for solving the Maxwell’s equation utilizing time harmonic excitation sources [8]. However, in many real world applications, the time harmonic source excitation is not suitable for image reconstruction. In this paper we extend the PBP method for electromagnetic tomographic imaging based on the time dependent Maxwell’s equations using wide band excitation sources. The contribution of our work is twofold. First, we provide a new framework for electromagnetic tomographic imaging using adjoint operators under broadband signal excitation in the time domain. Second, we develop an iterative yet convergent algorithm for reconstructing the dielectric profiles of the scattering medium for imaging purposes.

2. PROBLEM FORMULATION
The general time dependent Maxwell’s equations in an isotropic medium are given by

\[ \epsilon \frac{\partial E}{\partial t} = \nabla \times H + \sigma E = -J \] (1)

\[ \mu \frac{\partial H}{\partial t} + \nabla \times E = 0 \] (2)

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where the quantities $E$ and $H$ are the electric and magnetic field intensities. The materials parameters including electrical permittivity ($\epsilon$), magnetic permeability ($\mu$) and conductivity ($\sigma$) are independent of time. For simplicity, we assume that conductivity parameter $\sigma = 0$ and $\mu$ is a known constant. The quantity $J$ is the electric current density of an external charge and is considered as the external source. The equations (1) and (2) are defined on $\Omega \times [0,T]$, where $\Omega$ is the imaging region. For simplicity purposes, we assume that $\Omega$ is two-dimensional. Next, the initial conditions are given by

$$E(x,0) = 0 \quad (3)$$
$$H(x,0) = 0 \quad (4)$$

where $x = (x_1, x_2) \in \Omega$ is defined in a two dimensional space. For notational convenience, we further define a matrix operator

$$\Lambda = \begin{pmatrix} \frac{\partial}{\partial x} & -\nabla \times \frac{\partial}{\partial y} \\ \nabla \times & \mu \frac{\partial}{\partial y} \end{pmatrix} \quad (5)$$

Thus, the Maxwell’s equation (1)-(2) can be re-written as

$$\Lambda \begin{pmatrix} E \\ H \end{pmatrix} = \begin{pmatrix} -J \\ 0 \end{pmatrix} \quad (6)$$

The imaging problem is to reconstruct the dielectric profile of the scatterers given the equations (6) and the initial conditions (3) and (4).

Note that the imaging problem can also be modeled by the inverse problem of a two-dimensional wave propagation equation. Mathematically, the wave propagation equation (6) can be written as a nonlinear operator equation

$$\mathcal{R}_j(f) = g_j \quad (7)$$

where the symbol

$$f = [\epsilon, \mu]^T \quad (8)$$

stands for the material properties of the internal structure of the medium (or objects) to be reconstructed. For simplicity, the imaging problem we consider in this paper is to reconstruct the permittivity $\epsilon$ only. The operator

$$\mathcal{R}_j : L_2(\Omega) \to L_2(\Gamma \times [0,T]) \quad (9)$$

is a nonlinear mapping from the spaces $L_2(\Omega)$ onto $L_2(\Gamma \times [0,T])$, where $\Gamma$ denotes the boundary of the imaging region $\Omega$. The subscript $j$ refers to a projection direction or the $j$-th source excitation. The data $g_j$ is often a collection of the signals at the receiver antennas. The problem of interest is to find out the solution $f$, given (7). The problem in (7) is ill-posed and can be solved, in general, by iterative methods. One of the commonly used approaches is the adjoint method, which yields the update formula [5]

$$f^{k+1} = f^k + \omega \mathcal{R}_j^*(f^k) g_j - \mathcal{R}_j(f^k), \quad k = 0, 1, \cdots, (10)$$

where $f^0$ is the initial guess which may incorporate a priori knowledge of an estimate of the exact solution $f$. $k$ is the number of iterations. The symbol $\mathcal{R}_j^*(\cdot)$ is a locally uniformly bounded Fréchet derivative of $\mathcal{R}_j(\cdot)$ and $\mathcal{R}_j^*(\cdot)^*$ is the adjoint operator of $\mathcal{R}_j^*(\cdot)$.

### 3. THE IMAGING ALGORITHM

The imaging algorithm needs to repeat the scattering experiment for $j$-th excitation source, where $j = 1, \cdots, P$. At the $j$-th experiment, a broad band source is excited, the signal propagates through the medium. The scattered signals are recorded at the receiver antennas placed on the boundary of the imaging region. Given the measured data, the propagation and backpropagation method is utilized to reconstruct the image. The PBP method consists of 3 steps described as follows:

#### 3.1. Propagation

We use the finite difference time domain (FDTD) method in the forward solution to obtain 2-D wave field solution for an estimated dielectric profile. FDTD is a useful method for the simultaneous acquisition of multifrequency vector field data over the full bandwidth of interest [4]. The operator $\mathcal{R}_j$ is computed numerically within the imaging region $\Omega$. Thus, we obtain

$$\mathcal{R}_j(f^{(k)}) = u_j |_{\Gamma \times [0,T]} \quad (11)$$

where $u_j$ is the calculated data on the boundary $\Gamma$ given the estimated value $f^{(k)}$ at the $k$-th iteration. Note that in the computation, we use the MUR absorbing boundary condition [9].

#### 3.2. Calculate difference signal

The difference signal is given by

$$r_j = g_j - u_j |_{\Gamma \times [0,T]} \quad (12)$$

The difference signal $r_j$ will be backpropagate (i.e., time reversal re-transmission) to the same medium in order to calculate the increment value $\delta f^{(k)}$ to update the dielectric profile.

#### 3.3. Backpropagation

The backpropagation solver is obtained using the adjoint method. Once $f^{(k)}$ is determined at the $k$-th iteration, we calculate

$$f^{(k+1)} = f^{(k)} + \omega \delta f^{(k)} \quad (13)$$

where $\omega$ is a relaxation factor. Our goal is to compute the increment $\delta f^{(k)}$ at the $k$-th iteration using the adjoint operator $\mathcal{R}_j^*(r)$. The following theorem provides an explicit form for computing $\delta f^{(k)}$. For simplicity purposes, we omit $k$ in the following derivation.

**Theorem 1** The adjoint operator $\mathcal{R}_j^*(r) : L_2(\Gamma \times [0,T]) \to L_2(\Omega)$ is computed by

$$\mathcal{R}_j^*(r) = \int_0^T -\frac{\partial E}{\partial t} \cdot \mathcal{E} dt \quad (14)$$

where the quantities $E$ and $H$ are solved by the

$$-\epsilon \frac{\partial E}{\partial t} + \nabla \times H = \mathcal{R}_j(\epsilon) - g_j \quad (15)$$

$$-\mu \frac{\partial H}{\partial t} - \nabla \times E = 0 \quad (16)$$
subject to the initial conditions $E(x,T) = 0$ and $H(x,T) = 0$. Note $r_E$ is the $E$ component of the quantity $r$ defined in (12). The $H$ component $r_H$ is omitted because we only consider the reconstruction of the $E$ component.

**Proof.** The proof is given as follows. We let $\epsilon = \epsilon + \delta \epsilon$, $\mu = \mu + \delta \mu$. The corresponding electric and magnetic intensities can then be written as $E = E + \delta E$ and $H = H + \delta H$, respectively. Inserting the above expressions into (1) and (2) yields

\[(\epsilon + \delta \epsilon) \frac{\partial (E + \delta E)}{\partial t} - \nabla \times (H + \delta H) = -J \quad (17)\]

\[(\mu + \delta \mu) \frac{\partial (H + \delta H)}{\partial t} + \nabla \times (E + \delta E) = 0 \quad (18)\]

Next, subtract (1) and (2) from (17) and (18), respectively, we obtain

\[(\delta \epsilon) \frac{\partial E}{\partial t} + \frac{\partial (\delta E)}{\partial t} - \nabla \times \delta H = 0 \quad (19)\]

\[(\delta \mu) \frac{\partial H}{\partial t} + \mu \frac{\partial (\delta H)}{\partial t} + \nabla \times \delta E = 0 \quad (20)\]

Since we consider the reconstruction of $\epsilon$ only, and the parameter $\mu$ is constant, we let $\delta \mu = 0$, which yields

\[\mu \frac{\partial (\delta H)}{\partial t} + \nabla \times \delta E = 0 \quad (21)\]

Thus, we obtain

\[\Lambda \left( \frac{\delta E}{\delta H} \right) = \begin{pmatrix} -\delta \epsilon \frac{\partial E}{\partial t} \\ 0 \end{pmatrix} \quad (22)\]

Next we define the following initial conditions

\[\delta E(x,0) = 0, \quad \delta H(x,0) = 0 \quad (23)\]

\[E(x,T) = 0, \quad H(x,T) = 0 \quad (24)\]

Thus, we obtain

\[\int_0^T \int_\Omega \left( \epsilon \frac{\partial E}{\partial t} E + \mu \frac{\partial H}{\partial t} H \right) \, dx \, dt \quad (25)\]

\[= \int_0^T \int_\Omega \left( \epsilon \frac{\partial (\delta E)}{\partial t} E + \mu \frac{\partial (\delta H)}{\partial t} H \right) \, dx \, dt \quad (26)\]

Note that (26) holds due to the given initial conditions (23) and (24). Furthermore, due to the following identities

\[\int_\Omega (\nabla \times \delta E) \, H \, dx = \int_\Omega \delta E \, (\nabla \times H) \, dx \quad (27)\]

\[\int_\Omega (\nabla \times \delta H) \, E \, dx = \int_\Omega \delta H \, (\nabla \times E) \, dx \quad (28)\]

We obtain

\[\left\langle \Lambda \left( \frac{\delta E}{\delta H} \right), \frac{\partial E}{\partial t} \right\rangle_{L_2(\Omega \times [0,T])} \quad (29)\]

\[= \int_0^T \int_\Omega \left( \epsilon \frac{\partial (\delta E)}{\partial t} - \nabla \times \delta H \right) \, E \, dx \, dt + \int_0^T \int_\Omega \left( \nabla \times \delta E + \mu \frac{\partial (\delta H)}{\partial t} \right) \, H \, dx \, dt \quad (30)\]

\[= \int_0^T \left( \delta E \left( -\epsilon \frac{\partial E}{\partial t} + \nabla \times H \right) \right) \, dx \, dt \quad (31)\]

Hence

\[\Lambda^* = \begin{pmatrix} \nabla \times \left( -\epsilon \frac{\partial E}{\partial t} \right) \\ -\mu \frac{\partial E}{\partial t} \end{pmatrix} = -\Lambda \quad (32)\]

Note that the system (15)-(16) can be re-written as

\[\Lambda^* \left( \frac{\delta E}{\delta H} \right) = \left( \frac{r_E}{0} \right) = \left( \mathcal{R}_j(\epsilon) - g \right) \quad (33)\]

Thus

\[\left\langle \frac{\delta E}{\delta H}, \Lambda^* \left( \frac{\delta E}{\delta H} \right) \right\rangle_{L_2(\Omega \times [0,T])} \quad (34)\]

From (34) and (37), we obtain (14). By inserting (14) to (10), we obtain the expression for updating the dielectric parameter $\epsilon$. 

of a square shaped region (i.e., imaging region) with side length shown in Fig. 1. The object to be imaged is imbedded in a verify our results using a test phantom. The test phantom is In this section, we conduct numerical experiments to test and are placed on the boundary of the imaging areas. Bottom: Reconstructed image.

4. NUMERICAL EXPERIMENTS

In this section, we conduct numerical experiments to test and verify our results using a test phantom. The test phantom is shown in Fig. 1. The object to be imaged is imbedded in a square shaped region (i.e., imaging region) with side length of 9 cm (i.e., (−3 cm, 3 cm)). The antennas are placed on the four sides of the imaging region. The computational region is a square region with side length 21 cm (i.e., (−10.5 cm, 10.5 cm)). The object to be reconstructed is a dish like object with a diameter of 0.9 cm. We further assume that dielectric property function of the target is \( f(x) = 4\varepsilon_0 \) where \( \varepsilon_0 \) is the vacuum permittivity \( 8.85 \times 10^{-12} \) F/m. The permittivity value for the surrounding medium is \( \varepsilon_0 \). The imaging region is uniformly divided into small pixels of size 0.3 cm by 0.3 cm, which implies that the imaging area is discretized on a \( 140 \times 140 \) grid. As stated in the beginning of the paper, the goal of the tomographic imaging is to reconstruct the permittivity value \( f(x) \) within the region \( x \in \Omega \). There are a total of \( P = 30 \times 4 = 120 \) antenna sensors uniformly placed on the four sides of the the square. Each antenna element serves as the source for excitation and the receiver.

5. CONCLUSIONS

We developed an iterative propagation and back-propagation method for a nonlinear electromagnetic tomographic imaging problem in the time domain. We derived an explicit expression for calculating the dielectric profile of the target. Numerical examples demonstrate the success of the algorithm. Further research includes the convergence analysis of the proposed algorithm.

6. REFERENCES